

Algebraic Proof

(a) Prove that the difference of two odd numbers is even.

(b) Prove that $n^2 - 2 - (n - 2)^2$ is always an even number.

(c) Prove that the product of two odd numbers is always odd.

(d) Prove that the square of an even number is always even.

$$\begin{aligned} (a) \quad & 2n+1 - (2m+1) \\ & = 2n - 2m = 2(n-m) \end{aligned}$$

$$\begin{aligned} (b) \quad & n^2 - 2 - n^2 + 4n - 4 \\ & = 4n - 6 = 2(2n - 3) \end{aligned}$$

$$\begin{aligned} (c) \quad & (2n+1)(2m+1) \\ & = 4nm + 2m + 2n + 1 \\ & = 2(2nm + m + n) + 1 \end{aligned}$$

$$(d) \quad 2n \times 2n = 4n^2 = 2(2n^2)$$

$$(e) \quad 2n+3 - (2n+1) = 2$$

$$(f) \quad \frac{n+n+1+n+2}{3} = \frac{3n+3}{3} = n+1$$

$$\begin{aligned} (g) \quad & (n+1)^2 - n^2 \\ & = n^2 + 2n + 1 - n^2 \\ & = 2n + 1 \end{aligned}$$

$$\begin{aligned} (h) \quad & 7n + 56 + 5n - 20 \\ & = 12n + 36 = 12(n+3) \end{aligned}$$

$$\begin{aligned} (i) \quad & m^2 + 4m + 4 - m^2 - 12 \\ & = 4m - 8 = 4(m-2) \end{aligned}$$

$$\begin{aligned} (j) \quad & 2n+1 + 2n+3 + 2n+5 \\ & = 6n+9 = 3(2n+3) \end{aligned}$$

$$\begin{aligned} (k) \quad & (2n+1)^2 + (2m+1)^2 \\ & = 4n^2 + 4n + 1 + 4m^2 + 4m + 1 \\ & = 2(2n^2 + 2m^2 + 2n + 2m + 1) \end{aligned}$$

$$\begin{aligned} (l) \quad & 9n^2 + 6n + 1 - 9n^2 + 6n - 1 \\ & = 12n = 12(n) \end{aligned}$$

$$\begin{aligned} (m) \quad & n + n + 1 + n + 2 + n + 3 \\ & = 4n + 6 = 2(2n + 3) \\ & \text{even but not a multiple of 4} \end{aligned}$$

(e) Prove that the difference between any two consecutive odd numbers is always two.

(f) Prove that the mean of three consecutive integers is always the middle number.

(g) Prove that the difference between the squares of any two consecutive numbers is always odd.

(h) Prove that $7(n+8) + 5(n-4)$ is always a multiple of 12.

(i) Prove that $(m+2)^2 - m^2 - 12$ is always a multiple of 4.

(j) Prove that the sum of three consecutive odd numbers is always a multiple of three.

(k) Prove that the sum of the squares of any two positive odd integers is always even.

(l) Prove that $(3n+1)^2 - (3n-1)^2$ is always a multiple of 12 for all positive integer values of n .

(m) Prove that the sum of four consecutive integers is not divisible by four.